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# ELECTROMAGNETIC CHARACTERISTICS OF DOUBLY-PERIODIC MAGNETODIELECTRIC LAYER BOUNDED BY TWO UNIFORM MEDIA

Natalia V. Sidorchuk, Vladimir V. Yachin and Sergey L. Prosvirnin  
 Department of Calculus Mathematics, Institute of Radio Astronomy  
 4 Krasnoznamennaya Street, 61002 Kharkov, Ukraine  
 email: yachin@rian.kharkov.ua

## ABSTRACT

The problem of electromagnetic wave propagation in a doubly-periodic magnetodielectric layer bounded by two uniform infinite media is solved by new method based on the rigorous volume integro-differential equations of electromagnetics.

The Galerkin method is applied to reduce these equations to a set of second-order differential ones with constant coefficients in field functionals which contain information about geometry of the scattering structure. The special scheme of equation set solving is introduced in the case of thick layers to overcome usual numerical difficulties associated with the undesired exponential functions in the expressions. This method unifies the treatment of both TE- and TM-waves by replacing  $\varepsilon$  by  $\mu$ ,  $\mu$  by  $\varepsilon$ , E-components by H-components, H-components by E-components.

## METHOD

Formulation of the problem is as follows: from the region 2 ( $z < -h$ ) with complex relative permittivity  $\varepsilon_2$  and permeability  $\mu_2$ , a linearly-polarized plane electromagnetic wave is incident at an arbitrary angle  $\varphi$  on the double-periodic infinite layer (region 1) bordering the region 3 ( $z > 0$ ) with complex relative permittivity  $\varepsilon_3$  and permeability  $\mu_3$  (Fig.1).

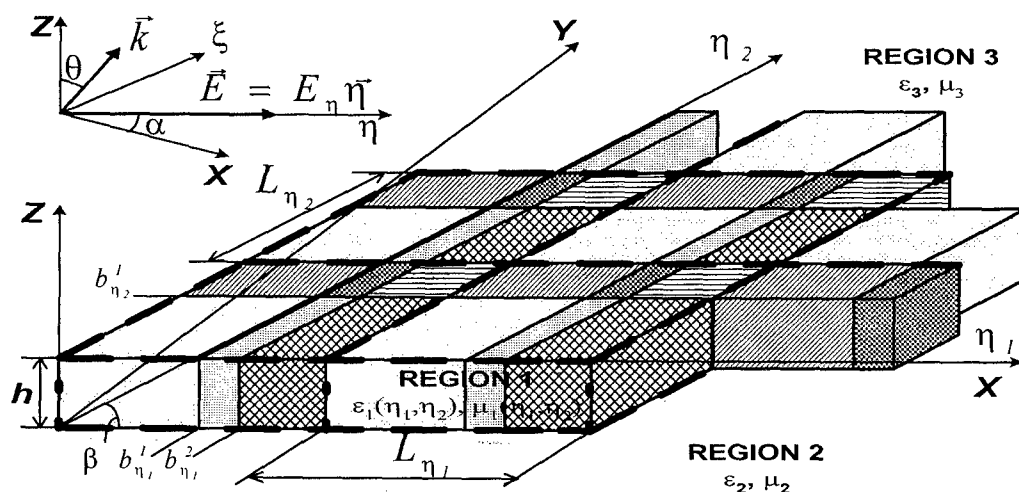


Figure 1. Geometry of the problem.

The periodic cell of the layer is an oblique-angle parallelepiped of arbitrary sizes along the  $\eta_1$  axis and the  $\eta_2$  axis,  $L_{\eta_1}$  and  $L_{\eta_2}$  are periodicities of the layer in the  $\eta_1$  and  $\eta_2$  direction, respectively. The parallelepiped is characterized by the complex relative permittivity  $\varepsilon_1(\eta_1, \eta_2)$  and permeability  $\mu_1(\eta_1, \eta_2)$  and has the thickness  $h$ . We suppose that the incident wave is TE-polarized and  $\alpha$  is the angle between the  $\eta_1$  axis and the electric field vector  $E$  lying in the plane of the layer and consider the field components in the  $(\eta, \xi, z)$  orthogonal coordinate system connected with the incident field polarization. We have to obtain the transmitted and reflected fields in the immediate vicinity of the layer. The form of the integro-differential equations for the electromagnetic field is taken from [1]:

$$\begin{aligned} E(\vec{r}) &= E_0(\vec{r}) + \frac{1}{4\pi} (\nabla\nabla + k_1^2) \int_{V'} \left( \frac{\varepsilon_1}{\varepsilon_2} - 1 \right) E(\vec{r}') G(\vec{r} - \vec{r}') d\vec{r}' + \\ &+ \frac{ik_1}{4\pi} \sqrt{\frac{\mu_2}{\varepsilon_2}} \cdot \nabla \times \int_{V'} \left( \frac{\mu_1}{\mu_2} - 1 \right) H(\vec{r}') G(\vec{r} - \vec{r}') d\vec{r}', \\ H(\vec{r}) &= H_0(\vec{r}) + \frac{1}{4\pi} (\nabla\nabla + k_1^2) \int_{V'} \left( \frac{\mu_1}{\mu_2} - 1 \right) H(\vec{r}') G(\vec{r} - \vec{r}') d\vec{r}' - \\ &- \frac{ik_1}{4\pi} \sqrt{\frac{\varepsilon_2}{\mu_2}} \cdot \nabla \times \int_{V'} \left( \frac{\varepsilon_1}{\varepsilon_2} - 1 \right) E(\vec{r}') G(\vec{r} - \vec{r}') d\vec{r}'. \end{aligned}$$

Here the Green's function is presented in the integral form,  $V$  is the scatterer volume,  $k_1 = k\sqrt{\varepsilon_2\mu_2}$  and  $k$  is a wave number. For beginning we suppose that the periods are partitioned into segments with constant material parameters. For each period segment numbered  $(k, l)$  we can write the notations  $b_{\eta_1}^{k-1} < \eta_{1k}' < b_{\eta_1}^k$ ,  $b_{\eta_2}^{l-1} < \eta_{2l}' < b_{\eta_2}^l$ ,  $\varepsilon_1(r_{kl}') = \varepsilon_{kl}$  and  $\mu_1(r_{kl}') = \mu_{kl}$ .

Following the algorithm given in [2] we present the field in each period segment as an expansion in terms of the spatial harmonics numbered  $(r, s)$  and act on the equation for these fields by the linear operator

$$\hat{A}_{pq}^{kl} F(\eta_1', \eta_2', z') = \frac{1}{L_{\eta_1} L_{\eta_2}} \int_{b_{\eta_1}^{k-1}}^{b_{\eta_1}^k} \int_{b_{\eta_2}^{l-1}}^{b_{\eta_2}^l} \left( \frac{\varepsilon_{kl}}{\varepsilon_2} - 1 \right) e^{-i(k_{\eta_1} + \frac{2\pi p}{L_{\eta_1}})\eta_1'} e^{-i(k_{\eta_2} + \frac{2\pi q}{L_{\eta_2}})\eta_2'} F(\eta_1', \eta_2', z') d\eta_1' d\eta_2'.$$

Thus we can obtain the set of linear differential equations for the field functionals. By summing these equations over all segments we can express the fields and field functionals for individual segment through the ones for another segment, e.g.:

$$E_{\eta pq}^{kl} = \frac{\varepsilon_{kT'}(\varepsilon_{kl} - 1)}{\varepsilon_{kl}(\varepsilon_{kT'} - 1)} (ax_{p-r}^k ay_{q-s}^l) (ax_{p-r}^{k'} ay_{q-s}^{l'})^{-1} E_{\eta pq}^{k'T'},$$

where

$$ax_{p-r}^k = \frac{1}{L_{\eta_1}} \int_{b_{\eta_1}^{k-1}}^{b_{\eta_1}^k} e^{-ix \frac{2\pi(p-r)}{L_{\eta_1}}} d\eta_1, \quad ay_{q-s}^l = \frac{1}{L_{\eta_2}} \int_{b_{\eta_2}^{l-1}}^{b_{\eta_2}^l} e^{-ix \frac{2\pi(q-s)}{L_{\eta_2}}} d\eta_2.$$

Next we solve the equation set for the field functionals and thus for the field components (the procedure in paper [2]). Using the extinction theorem at the last stage we express the internal fields of the structure (region 1 and 3) through the incident wave field. Upon separation the equation set to be solved onto the subsets including the exponential functions with a positive and negative real part of eigenvalues, we apply the iterative procedure developed for calculations of characteristics of scattering from a very thick lossy doubly-periodic magnetodielectric layer in a wide frequency range.

## RESULTS

Presented in Fig.2 are the results of the transmission coefficient calculation for the periodic structure composed of the array of square parallelepipeds with  $\varepsilon = 3+0.01i$ ,  $\mu = 1$ , lying on the half-space with  $\varepsilon_3 = 3$ ,  $\mu_3 = 1$ ,  $h/L_{\eta_1} = 10$ ,  $b_{\eta_1}/L_{\eta_1} = b_{\eta_2}/L_{\eta_2} = 0.5$ ,  $L_{\eta_1} = L_{\eta_2} = L_{\eta}$ ,  $\alpha = 25^\circ$ ,  $\varphi = 0.001^\circ$ . The example demonstrates the iterative procedure stability for very thick doubly-periodic structures.

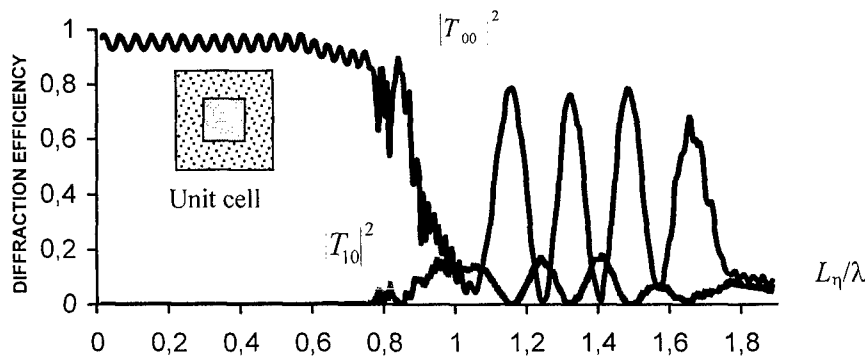


Figure 2. Transmission of the normally incident plane wave through the thick doubly-periodic array lying on the half-space.

Our results are in a good agreement with data concerned thick periodic structures and presented by other authors (see, for example, paper [3], where the one-periodic grating of rectangular rods with  $h/L = 4$  was considered).

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